Formalizing the Lazy Intruder in Isabelle:
Towards Formalized Protocol Compositionality Results

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Relative Soundness

Examples from [Almousa et al., 2015]:

Theorem (Typing result)

If P is a type-flaw resistant protocol and has an attack, then P has a well-typed attack

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Example: type-flaw attack; $[K_{AB} \mapsto (M, A, B)]$

$$\begin{aligned} A \rightarrow B: &\quad M, A, B, \mathrm{scrypt}(k, (M, A, B)) \\ B \rightarrow A: &\quad M, \mathrm{scrypt}(k, (N_A, K_{AB})) \end{aligned}$$

Attack: Unifying K_{AB} and (M, A, B) enables the intruder to send the second message $\Longrightarrow$ K_{AB} becomes known

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The proofs of these theorems depend on the lazy intruder

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- Constraints on Dolev-Yao style intruder deduction
- Is sound, complete, and terminating:
  ($\exists$simple constraint ψ. $\phi \leadsto^* \psi \land \mathcal{I} \models \psi$) iff $\mathcal{I} \models \phi$,
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Example constraint C:

$$\{pk, \mathrm{crypt}(pk, secret)\} \vdash \mathrm{crypt}(pk, X) \wedge \{pk, \mathrm{crypt}(pk, secret), h(X)\} \vdash Y$$

one solution is the following:

$$C \rightsquigarrow \{pk, \mathrm{crypt}(pk, secret), h(secret)\} \vdash Y$$

Constraint reduced to a simple (always satisfiable) constraint

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This is done as follows

- Any attack can be seen as a solution to a constraint
- Since the lazy intruder is complete, it will find a solution
- Show that all reduction steps preserve some invariant
  - e.g. no ill-typed instantiations of variables
- Show that the preservation implies the original property
- Thus: if there is an attack, then there is one where the solution has the property

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Example: Part of a typing result proof [Almousa et al., 2015]:

(Equation). For the *(Unify)* rule, we proceed by cases of s and t:

- If both s and t are atomic, then s and t cannot be variables, so the above property is preserved trivially, simply because they must be the same constant.
- If both are composed, then $\sigma(s) = \sigma(t)$ and there exist $u, v \in SMP$ and ϑ_1, ϑ_2 such that $\vartheta_1(u) = s$ and $\vartheta_2(v) = t$. Then, $\sigma(\vartheta_1(u)) = \sigma(\vartheta_2(v))$ and $\Gamma(u) = \Gamma(v) = \Gamma(s) = \Gamma(t)$ as the protocol is type-flaw-resistant, and so σ is well-typed.
- If t is variable, then it is simple and we proved earlier that if it has an ill-typed solution, then it also has a well-typed one.

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- ... but requires a huge time investment for proof development
  - Informal arguments not accepted
- Simpler definitions leading to simpler proofs can be useful

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Contributions (finished, modulo some details):

- Formalization of a lazy intruder in Isabelle/HOL
- Formalization of a typing result based on the lazy intruder
- Work towards formalization of a parallel compositionality result based on the typing result

Simplification: Constraints As Strands

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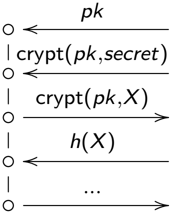
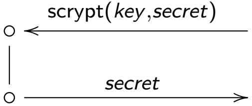
An easier representation:



Intruder knowledges implicit, monotonically growing

Simplification: Analysis As Protocol Steps



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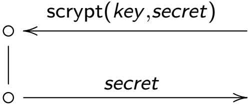

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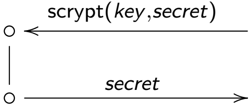

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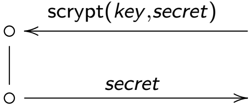

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- Completeness proof based on traversing or restricting derivation trees

Example: Informal Reasoning

Part of proving completeness of a lazy intruder constraint system [Cortier et al., 2007]

Definition 16 (simple) We say that a proof π is *simple* if

1. any subproof of π is left-minimal,
2. a composition rule of the form $\dfrac{u_1 \quad u_2}{u}$ is not followed by a decomposition rule leading to u_1 or u_2,

Lemma 2 *Let $\mathcal{C}$ be an unsolved constraint system, θ be a solution of $\mathcal{C}$ and $T_i \Vdash u_i$ be a minimal unsolved constraint of $\mathcal{C}$. Let u be a term. If there is a simple proof of $T_i\theta \vdash u$ having the last rule an axiom or a decomposition then there is $t \in St(T_i) \setminus \mathcal{X}$ such that $t\theta = u$.*

Example: Informal Reasoning

Part of proving completeness of a lazy intruder constraint system [Almousa et al., 2015]

– If the node is an application of the (*Decompose*) rule, then consider the ground term t that is being decomposed in the derivation proof for $\mathcal{I}(t_i)$. We first consider different cases depending on how t is derived:

- If t is obtained by a decomposition step itself, then we regress to the respective term being decomposed, and we do so until we hit a term that is not obtained by decomposition. By the previous cases, this cannot

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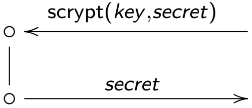

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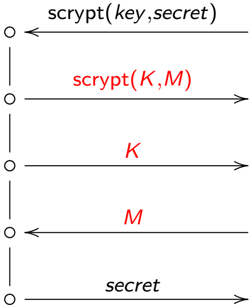
Idea: Analysis as protocol steps

Example: Analysis As Protocol Steps

With explicit analysis



Only finitely many analyzable terms given a finite intruder knowledge
Ill-typed unification between variables possible!

Example: Analysis As Protocol Steps

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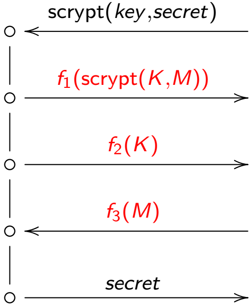

Messages wrapped in formats to prevent ill-typed unification between variables (needed for typing result)

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$$\mathcal{I}(trp(\mathcal{M})) \subseteq trp(\mathcal{I}(\mathcal{M}))$$

For well-formed constraints:

Theorem

$trp(\mathcal{I}(\mathcal{M})) \vdash_c t$ if and only if $\mathcal{I}(trp(\mathcal{M})) \vdash_c t$

Conclusion

Formalization of the lazy intruder in Isabelle/HOL

- With simplifications
- Soundness, completeness, termination proved

Relative soundness typing theorem formalized

- "exists attack $\Longrightarrow$ exists well-typed attack"

Future work (compositionality!)

- Formalize parallel compositionality theorem of [Almousa et al., 2015]
- Formalize and unify other results based on the lazy intruder, e.g. [Cortier et al., 2007]